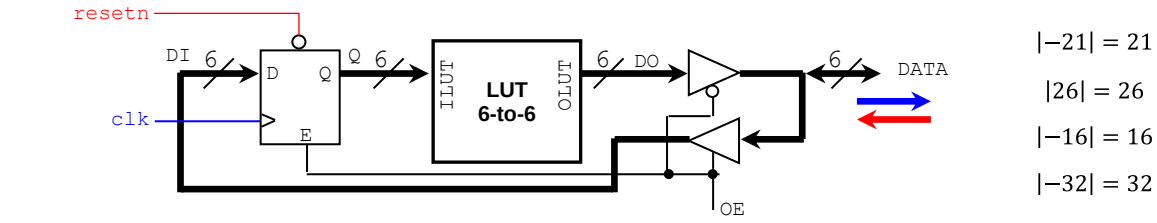
# Solutions - Final Exam

(April 27th @ 7:00 pm)

Presentation and clarity are very important! Show your procedure!

## PROBLEM 1 (11 PTS)

- Given the following circuit, complete the timing diagram.
  The LUT 6-to-6 implements the following function: $OLUT = |ILUT|$ (absolute value), where $ILUT$ is a 6-bit signed (2C ) number, and $OLUT$ is a 6-bit unsigned number.
  For example $ILUT = -29 = 100011_2 \rightarrow OLUT = |-29| = 29\ (011101_2)$

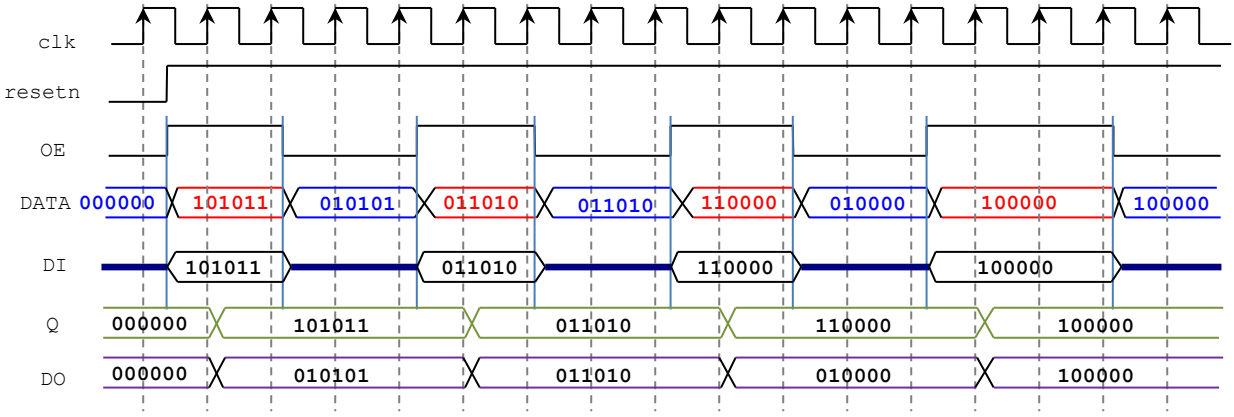



$|-21| = 21$

$|26| = 26$

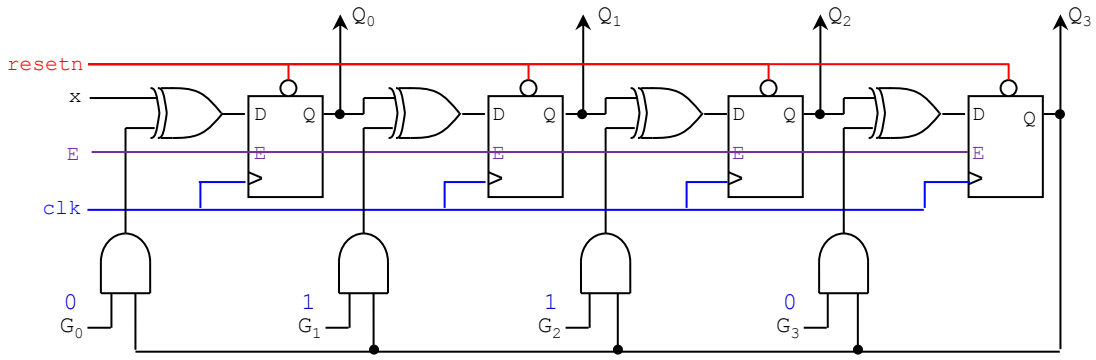
$|-16| = 16$

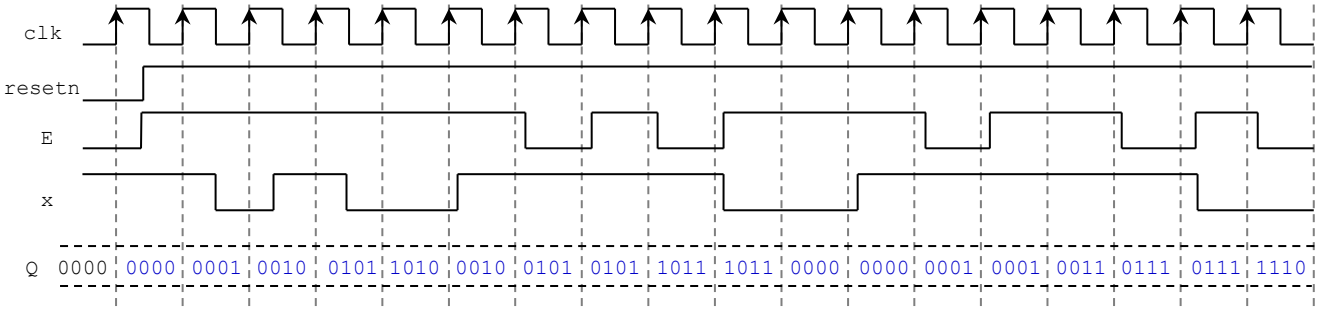
$|-32| = 32$




## PROBLEM 2 (12 PTS)

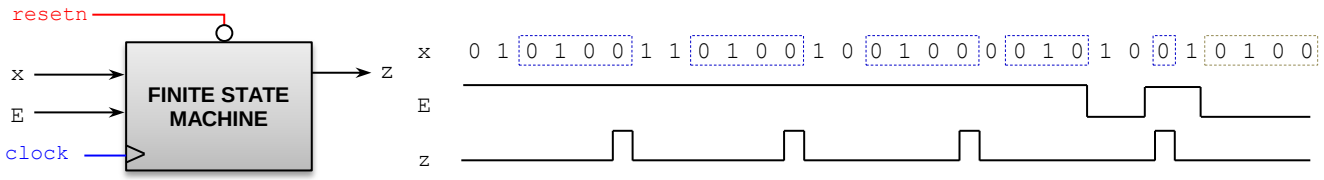
- Complete the timing diagram of the following circuit. $G = G_3G_2G_1G_0 = 0110$, $Q = Q_3Q_2Q_1Q_0$



# PROBLEM 3 (24 PTS)

- Sequence detector: The machine generates $z = 1$ when it detects the sequence $0100$. Once the sequence is detected, the circuit looks for a new sequence.
- The signal E is an input enable: It validates the input $x$, i.e., if E=1, $x$ is valid, otherwise $x$ is not valid.




- Draw the State Diagram (any representation) of this circuit with inputs E and x and output z. (7 pts)
- Complete the State Table and the Excitation Table (8 pts.)
- Provide the excitation equations and the Boolean output equation (simplify your circuit: K-maps or Quine-McCluskey).
- Sketch the circuit. (3 pts)
- Which type is this FSM? ~~(Mealy)~~ (Moore) Why? _____

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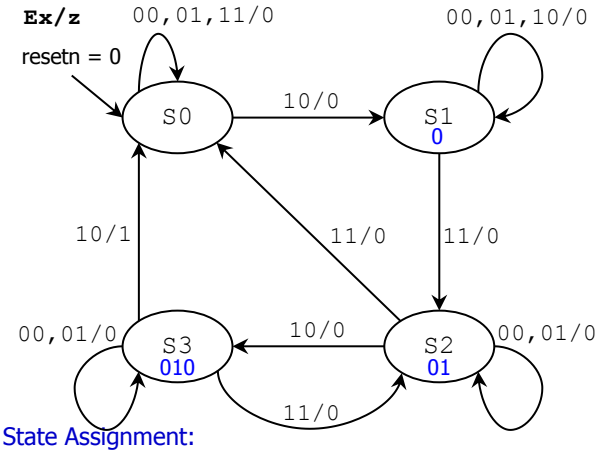
- *State Diagram, State Table, and Excitation Table*:




State Assignment:
S0: Q=00 S1: Q=01
S2: Q=10 S3: Q=11

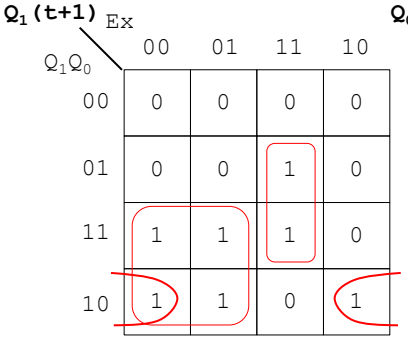
| E | x | PRESENT STATE | NEXT STATE | z |
|---|---|---|---|---|
| 0 | 0 | S0 | S0 | 0 |
| 0 | 0 | S1 | S1 | 0 |
| 0 | 0 | S2 | S2 | 0 |
| 0 | 0 | S3 | S3 | 0 |
| 0 | 1 | S0 | S0 | 0 |
| 0 | 1 | S1 | S1 | 0 |
| 0 | 1 | S2 | S2 | 0 |
| 0 | 1 | S3 | S3 | 0 |
| 1 | 0 | S0 | S1 | 0 |
| 1 | 0 | S1 | S1 | 0 |
| 1 | 0 | S2 | S3 | 0 |
| 1 | 0 | S3 | S0 | 1 |
| 1 | 1 | S0 | S0 | 0 |
| 1 | 1 | S1 | S2 | 0 |
| 1 | 1 | S2 | S0 | 0 |
| 1 | 1 | S3 | S2 | 0 |

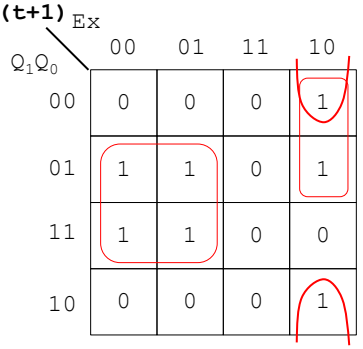
| E | x | $Q_1Q_0(t)$ | $Q_1Q_0(t+1)$ | z |
|---|---|---|---|---|
| 0 | 0 | 0 0 | 0 0 | 0 |
| 0 | 0 | 0 1 | 0 1 | 0 |
| 0 | 0 | 1 0 | 1 0 | 0 |
| 0 | 0 | 1 1 | 1 1 | 0 |
| 0 | 1 | 0 0 | 0 0 | 0 |
| 0 | 1 | 0 1 | 0 1 | 0 |
| 0 | 1 | 1 0 | 1 0 | 0 |
| 0 | 1 | 1 1 | 1 1 | 0 |
| 1 | 0 | 0 0 | 0 1 | 0 |
| 1 | 0 | 0 1 | 0 1 | 0 |
| 1 | 0 | 1 0 | 1 1 | 0 |
| 1 | 0 | 1 1 | 0 0 | 1 |
| 1 | 1 | 0 0 | 0 0 | 0 |
| 1 | 1 | 0 1 | 1 0 | 0 |
| 1 | 1 | 1 0 | 0 0 | 0 |
| 1 | 1 | 1 1 | 1 0 | 0 |

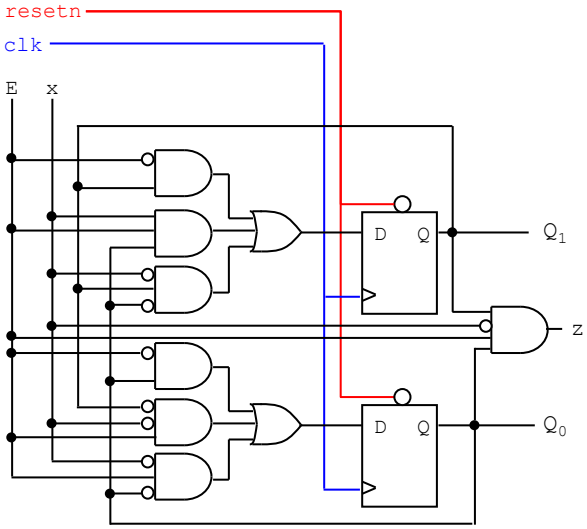
This is a Mealy Machine. The output $z$ depends on the input as well as on the present state.

- *Excitation equations, minimization, and circuit implementation*:

$$Q_1(t+1) \leftarrow \bar{E}Q_1 + ExQ_0 + \bar{x}Q_1\overline{Q_0}$$
$$Q_0(t+1) \leftarrow \bar{E}Q_0 + E\bar{x}\overline{Q_1} + E\bar{x}\overline{Q_0}$$
$$z = E\bar{x}Q_1Q_0$$

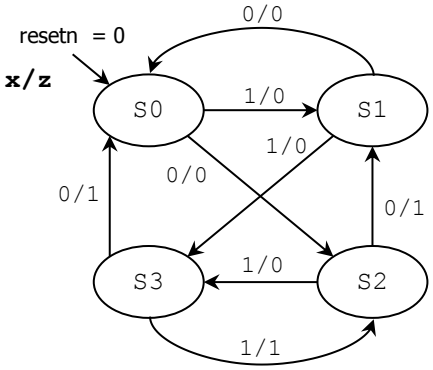


# PROBLEM 4 (22 PTS)

a) Given the following State Machine Diagram: (11 pts)



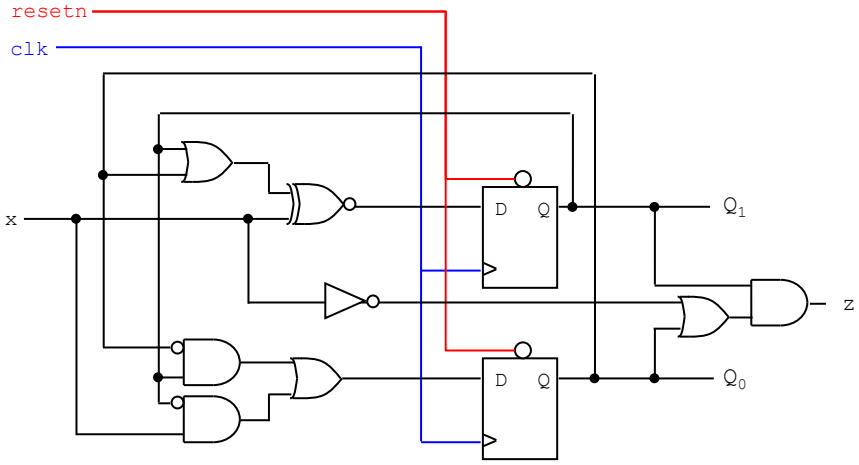

- ✓ Provide the State Table and the Excitation Table (4 pts.)
- ✓ Get the excitation equations and the Boolean equation for z. (3 pts.)
  Use S0 (Q=00), S1 (Q=01), S2 (Q=10), S3 (Q=11) to encode the states.
- ✓ Sketch the Finite State Machine circuit. (3 pts.)
- ✓ Which type is this FSM? ~~(Mealy)~~ (Moore)

| x | PRESENT STATE | NEXT STATE | z |
|---|---|---|---|
| 0 | S0 | S2 | 0 |
| 0 | S1 | S0 | 0 |
| 0 | S2 | S1 | 1 |
| 0 | S3 | S0 | 1 |
| 1 | S0 | S1 | 0 |
| 1 | S1 | S3 | 0 |
| 1 | S2 | S3 | 0 |
| 1 | S3 | S2 | 1 |

| x | $Q_1Q_0(t)$ | | $Q_1Q_0(t+1)$ | | z |
|---|---|---|---|---|---|
| | PRESENT STATE | | NEXTSTATE | | |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |

$Q_1(t+1) \leftarrow \overline{(Q_1 + Q_0) \oplus x}$
$Q_0(t+1) \leftarrow x\overline{Q_1} + Q_1\overline{Q_0}$
$z = \bar{x}Q_1 + Q_1Q_0$

*State Assignment*:
S0: Q=00 S1: Q=01
S2: Q=10 S3: Q=11

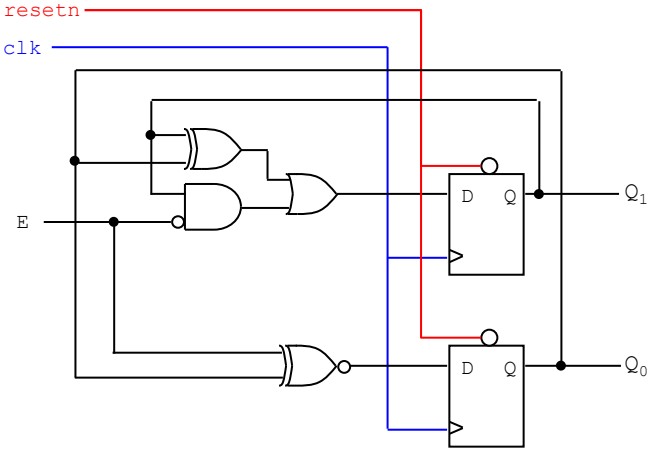



b) A synchronous circuit (with *resetn* and *clock*), is described by these excitation equations (E is a synchronous input): (11 pts.)

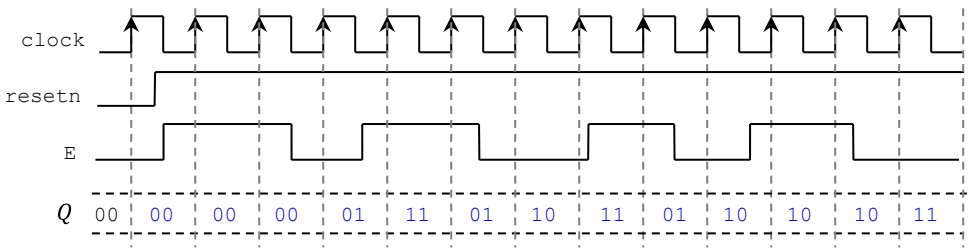
$$Q_1(t+1) \leftarrow Q_1(t).\overline{Q_0(t)} + \bar{E}.Q_1(t) + \overline{Q_1(t)}.Q_0(t)$$
$$Q_0(t+1) \leftarrow E.Q_0(t) + \bar{E}.\overline{Q_0(t)}$$

- ✓ With flip flops and logic gates, sketch the circuit.

$Q_1(t+1) \leftarrow Q_1(t) \oplus Q_0(t) + \bar{E}.Q_1(t)$
$Q_0(t+1) \leftarrow \overline{E \oplus Q_0(t)}$



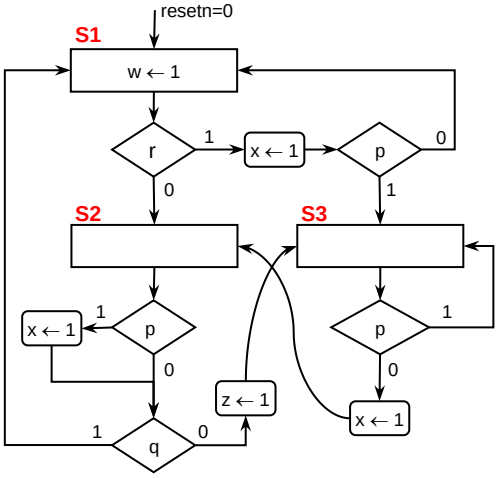

- ✓ Complete the timing diagram. $Q = Q_1Q_0$ (Tip: get the excitation table) (6 pts)



# PROBLEM 5 (13 PTS)

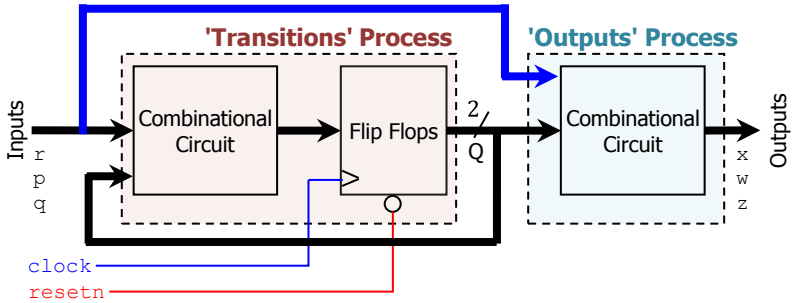
- Draw the State Diagram (in ASM form) of the FSM whose VHDL description in shown below. (7 pts.)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
   port ( clk, resetn: in std_logic;
          r, p, q: in std_logic;
          x, w, z: out std_logic);
end circ;
```




```
architecture behavioral of circ is
   type state is (S1, S2, S3);
   signal y: state;
begin
  Transitions: process (resetn, clk, r, p, q)
  begin
     if resetn = '0' then y <= S1;
     elsif (clk'event and clk = '1') then
        case y is
          when S1 =>
            if r = '0' then
               y <= S2;
            else
               if p = '1' then y <= S3; else y <= S1; end if;
            end if;

          when S2 =>
            if q = '1' then y <= S1; else y <= S3; end if;

          when S3 =>
            if p = '1' then y <= S3; else y <= S2; end if;
        end case;
     end if;
  end process;

  Outputs: process (y, r, p, q)
  begin
      x <= '0'; w <= '0'; z <= '0';
      case y is
         when S1 => w <= '1';
                    if r = '1' then x <= '1'; end if;

         when S2 => if p = '1' then x <= '1'; end if;
                    if q = '0' then z <= '1'; end if;

         when S3 => if p = '0' then x <= '1'; end if;
      end case;
  end process;
end behavioral;
```

- The figure shows an FSM model representing the circuit described in VHDL.
  The state (signal 'y' in the VHDL code) is represented by the bits $Q_1$ and $Q_0$.




✓ If we use S1 (Q=00), S2 (Q=01), S3 (Q=10) to encode the states, what is the Boolean equation for $w$? (2 pts.)
$w = \overline{Q_1(t)}.\overline{Q_0(t)}$

✓ Circle the correct answer: (4 pts.)

| | | |
|---|---|---|
| ▫ The 'Outputs' process outputs depend on clock and resetn? | TRUE | ~~FALSE~~ |
| ▫ The relationship between [r,p,q, present state] and [next state] is described by: | ~~Transitions Process~~ | Outputs Process |
| ▫ The relationship between [r,p,q, present state] and [outputs x,w,z] is described by: | Transitions Process | ~~Outputs Process~~ |
| ▫ Is this a Mealy or a Moore FSM? | Moore | ~~Mealy~~ |

# PROBLEM 6 (18 PTS)

- Complete the timing diagram of the following digital circuit that includes an FSM (in ASM form) and a Datapath circuit.
  - ✓ The behavior (on the clock tick) of the generic register is as follows:

4-bit register: If E=0, the output is kept

```
if E = 1 then
   if sclr = 1 then
      Q ← 0
   else
      Q ← D
  end if;
end if;
```